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On Modeling the Volatility of Nigeria's Foreign Exchange (Bureau-de-change) Using Hybrid ARIMA-ARCH and Hybrid ARIMA-GARCH Models

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***Abstract**: This study is aimed at modeling and forecasting the volatility of monthly foreign exchange (bureau de change) from the Naira to the US Dollar in Nigeria. Time series plot analyses were used to observe the pattern and fluctuations in the volatility of bureau-de-change from January 2004 to December 2018 obtained from the CBN Statistical Bulletin. It was observed that the series has a trend pattern; the series became stationary after the first difference. Several ARIMA-ARCH/GARCH models were fitted; the best model among hybrid ARIMA-ARCH models is ARIMA/ARCH (2, 1, 1; 3), while the best model among hybrid ARIMA-GARCH models is ARIMA-GARCH (1, 1, 1; 1). These best models were used both in modeling and forecasting; a 12-month step-ahead forecast for the exchange rates was employed. The forecast shows a decline in the value of the naira.*

***Keywords:** Arima, Hybrid, Arch, Garch, Cbn, Volatility.*

1.0 Introduction

A statistical measure of the dispersion of returns for a given security or market index is called volatility. Volatility can either be measured by using the standard deviation or variance between returns from that same security or market index. On the other hand, the higher the volatility, the riskier the security. A variable in the option pricing formula shows the extent to which the returns of the underlying asset will fluctuate between now and the option's expiration. Volatility, expressed as a percentage coefficient within option pricing formulas, arises from daily trading activities. How volatility is measured will affect the value of the coefficient used. Volatility refers to the amount of uncertainty or risk about the size of changes in a security's value. A higher volatility means the security's value can potentially be spread out over a larger range of values. This means that the price of a security can change dramatically over a short period of time in either direction. A lower volatility means that a security's value does not fluctuate dramatically but changes in value at a steady pace over a period of time. The foreign exchange market is volatile in nature, which makes it very dramatic sometimes.

The most important problem of forecasting depends on how appropriate the method is used to fit the time series data, which also depends on how the data is (nature of the data). In this study, hybrid models of time series were used to fit the foreign exchange (bureau-de-change) data.

2.0 Methodology

Because of some drawbacks and limitations on ARIMA and ARCH/GARCH family models, some statisticians came up with the idea of mixing the two families to obtain a hybrid version where the parameters of each model remain intact.

(i) Hybrid ARIMA- ARCH: This is combination of ARIMA and ARCH Models

$$ARIMA + ARCH = \emptyset_1 y_{t-1} + \ldots + \emptyset_p y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q} + \propto_0 + \propto_1 \varepsilon^2_{t-1} + \ldots + \propto_q \varepsilon^2_{t-q}$$

Where $\varepsilon_t \sim N(0,\sigma_t^2)$, p and q are the number of autoregressive terms and the number of lagged forecast errors, respectively. while for ARCH terms $\propto_0 > 0$, and $\propto_i >= 0$, $i > 0$. To assure $\{\sigma^2{}_t\}$ is asymptotically stationary random sequence, we can assume that $\propto_1 + \propto_2 + \ldots \propto_q < 1$.

(ii) Hybrid ARIMA- GARCH: This is combination of ARIMA and GARCH Models

$$ARIMA + GARCH = \emptyset_1 y_{t-1} + \ldots + \emptyset_p y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q} + \propto_0 + \propto_1 \varepsilon^2_{t-1} + \ldots + \propto_q \varepsilon^2_{t-q} + \beta_j h_{t-1} + \ldots + \beta_p h_{t-p}$$

Where $\varepsilon_t \sim N(0,\sigma_t^2)$, p and q are the number of autoregressive terms and the number of lagged forecast errors, respectively. While, the GARCH part are; h_t is the conditional variance, h_{t-j} is the past conditional variance, ε^2_{t-i} past squared residual return and $\propto_0 > 0$, $\propto_i >= 0$, $\beta_j >= 0$.

3.0 Data Analyses and Results

In this paper, time series data on monthly exchange rates (bureau-de-change) from foreign exchange markets for the period of fifteen years was collected. The data were collected from January 2004 to December 2018, which created 180 observations. Most of the computational work is carried out using R statistical software. The data obtained were analyzed to check if the data is stationary or has a unit root using the Augmented Dickey Fuller test (ADF), Phillips Peron (PP), and Kiatkowski-Phillips-Schmidt-Shin (KPSS) tests. The autocorrelation and partial autocorrelation functions of the data were also plotted to confirm the statuary of its stationarity. However, the time series plot that displays the observations on the y-axis against equally spaced time intervals on the x-axis used to evaluate patterns and behavior in data over time is displayed in Figure1below:

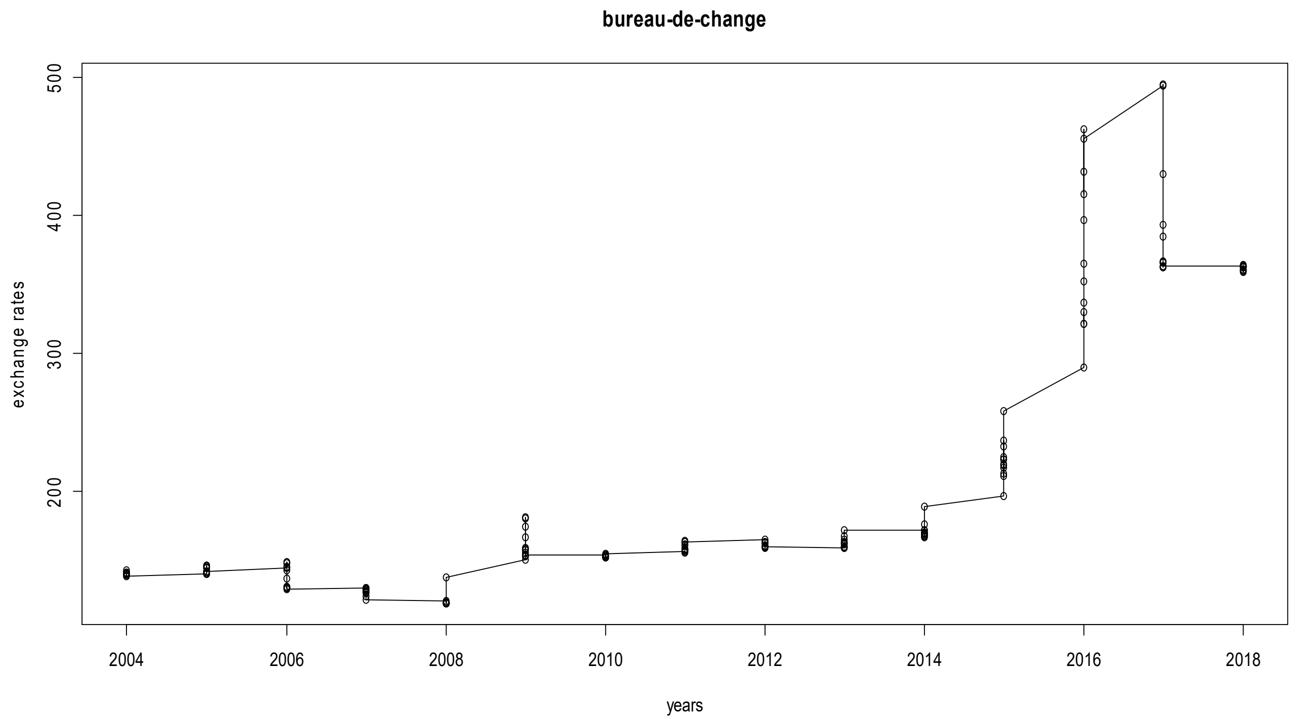

Figure 1 Bureau-de-change Rates

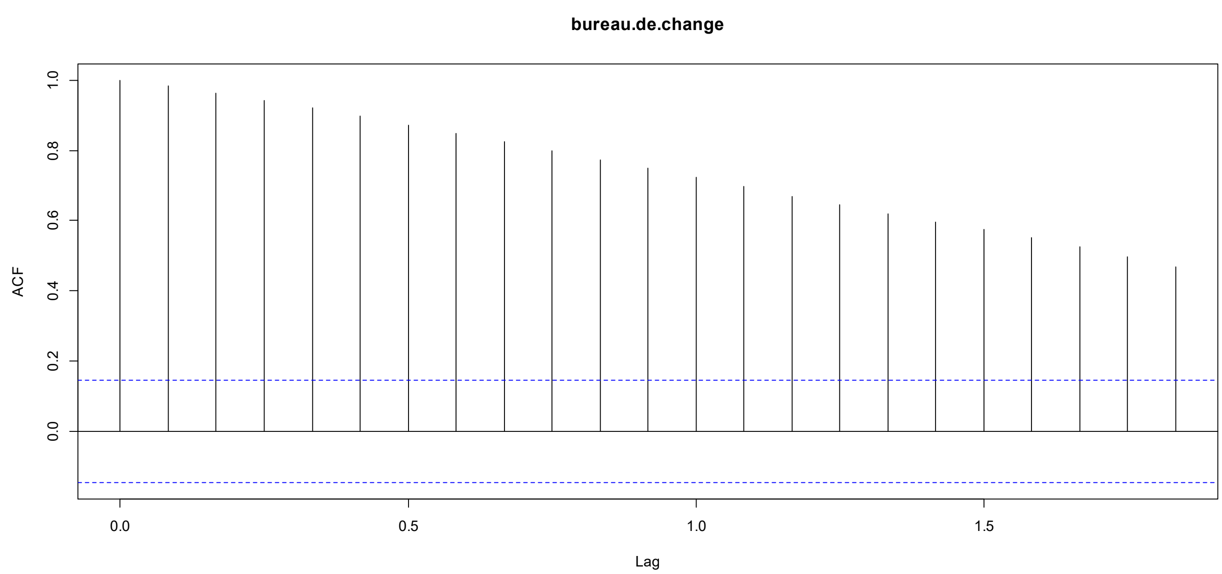

Fig. 2: ACF plot for Bureau-de-change Rates

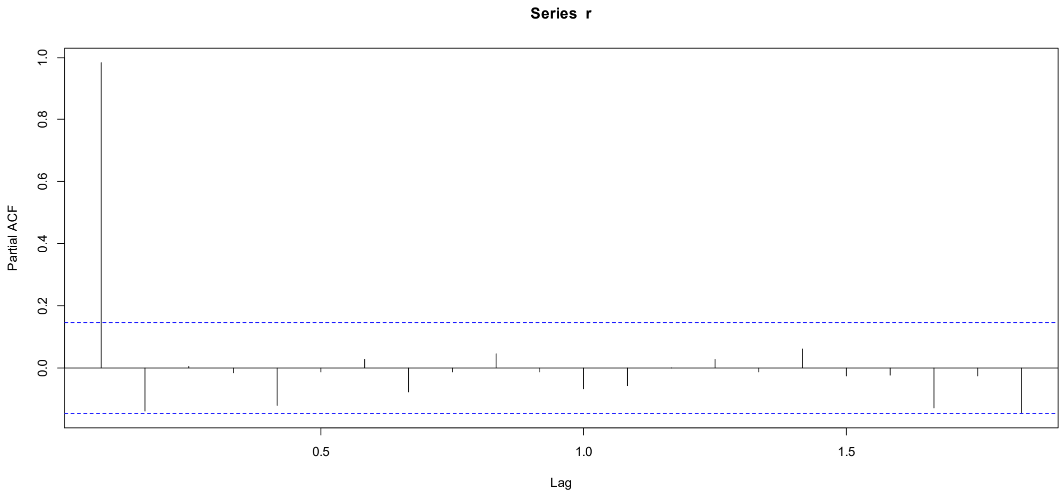

Fig. 3: PACF plot for Bureau-de-change Rates

Figures 2 and 3 show the correlograms of bureau-de-change exchange rates. The charts show that the bureau-de-change data series is not stationary. Stationarity is a condition regarding the independence of the data in the time series over time. In this instance, the reason the data is not stationary is that each time point is clearly dependent on previous time steps. This is most clearly seen in the ACF chart of the data. Note also that the main feature of the correlellogram for data to be stationary is that the autocorrelation tends towards zeros as the lags increase. This is not found in this correlogram. The solution to this problem is to divide the data and then recalculate the ACF and PACF on this new time series. Differencing is not a complicated-sounding term but is really just creating a new series by subtracting the previous data point from the current one. Mathematically:

$$X_t = Y_t - Y_{t-1}$$

Note that the 'difference data set' has one fewer observation than the original data set. If these new data are still not stationary, the process can be repeated until stationary data is obtained; typically, a non-stationary data set only needs to be changed one or two times to obtain a stationary data set.

Table 1: Test for unit root of bureau-de-change exchange rate

Test statistic	Values	Lag order	P-value	Hypothesis (H_0)	Decision	Remark
ADF	-2.0121	5	0.5711	Unit root	Accept H_0	Not stationary
PP	-5.5785	4	0.7954	Unit root	Accept H_0	Not stationary
KPSS	2.4764	4	0.01	Stationary	Reject H_0	Not stationary

Table 1 above shows ADF, PP and KPSS test statistic of -2.0121, -5.5785 and 2.4764 with p-values 0.5711, 0.7954 and 0.01 respectively, where ADF and PP has values which are greater than the critical value of 0.05 while KPSS has a value less than the critical value. We accept null hypothesis of having unit root series for ADF and PP and reject a null hypothesis of being a stationary series for KPSS. Indeed, the three tests confirm that the data series is not stationary.

It clears the time series plot of the bureau-de-change data series, the ACF and PACF with their correlellogram and stationarity tests suggests that the data need to be transformed or differenced since it is confirm to have a unit root.

Table 2: Test for unit root of bureau-de-change exchange rate after the first difference.

Test statistic	Values	Lag order	P-value	Hypothesis (H_o)	Decision	Remark
ADF	-5.1158	5	0.01	Unit root	Reject H_o	Stationary
PP	-125.31	4	0.01	Unit root	Reject H_o	Stationary
KPSS	0.1978	4	0.1	Stationary	Accept H_o	Stationary

Table 2 presents the stationarity tests for the differenced bureau-de-change rate over the period of investigation with a null hypothesis of a unit root against an alternative hypothesis of a level of stationarity for ADF and PP and vice versa for KPSS. The p-values of 0.01 both for ADF and PP are less than the 5% level of significance, while the p-value of 0.1 for KPSS is greater than the 5% level of significance, which indicates that the null hypothesis of having a unit root series should be rejected in favor of the alternative of being stationary and vice versa for KPSS. Indeed, the data is stationary after the first difference; hence, fitting and forecasting of the series proceeded.

Table 3: Fitting hybrid ARIMA-ARCH on differenced bureau-de-change.

Model	ARIMA(1,1,1)+ ARCH(1)	ARIMA(1,1,2)+ ARCH(2)	ARIMA(2,1,1)+ ARCH(3)	ARIMA(2,1,2)+ ARCH(4)
AR(1)	0.3426	0.0224	0.4156	0.4558
AR(2)	-	-	-0.1091	-0.0892
MA(1)	0.0880	0.0501	-0.0438	-0.0847
MA(2)	-	0.2101	-	-0.0370
OMEGA	9.1566	5.6188	5.3827	5.4214
ALPHA(1)	1.0000	0.3916	0.2879	0.2857
ALPHA(2)	-	0.7317	0.1251	0.1260
ALPHA(3)	-	-	0.4257	0.4176
ALPHA(4)	-	-	-	0.00000001
AIC	6.2298	5.8718	*5.7597*	5.7972
BIC	6.3010	5.9787	*5.8843*	5.9574
SIC	6.2288	5.8697	*5.7568*	5.7924

From table 3, hybrid ARIMA-ARCH (2, 1, 1; 3) has the smallest AIC, BIC, and SIC, and it is therefore regarded as the best model for fitting the bureau-de-change exchange rate. Furthermore, the estimated coefficient values for all ARIMA-ARCH (p, d, and q) strictly conform to the bounds of the parameters, between -1 and 1. This has also made the model stationary.

Table 4: Fitting hybrid ARIMA-GARCH on differenced bureau-de-change exchange rates

Model	ARIMA(1,1,1)+ GARCH(1,1)	ARIMA(1,1,2)+ GARCH(1,2)	ARIMA(2,1,1)+ GARCH(2,1)	ARIMA(2,1,2)+ GARCH(2,2)
AR(1)	-0.1168	0.2860	-0.4481	-0.2568
AR(2)	-	-	0.1598	0.2587
MA(1)	0.4236	0.0104	0.8060	0.5690
MA(2)	-	-0.1721	-	-0.1891
OMEGA	3.2713	3.3541	3.9605	4.0050
ALPHA(1)	0.5870	0.5929	0.3002	0.3109
ALPHA(2)	-	-	0.4240	0.4436
BETA(1)	0.3960	0.3823	0.2550	0.2403
BETA(2)	-	0.00000001	-	0.00000001
AIC	*5.8074*	5.8391	5.8082	5.8247
BIC	*5.8964*	5.9637	5.9328	5.9850
SIC	*5.8050*	5.8362	5.8053	5.8199

From Table 4, hybrid ARIMA-GARCH (1,1,1,1) has the smallest AIC, BIC, and SIC, and it is therefore regarded as the best model for fitting the bureau-de-change exchange rate. Furthermore, the estimated coefficient values for all ARIMA-GARCH (p,d,q) strictly conform to the bounds of the parameters, between -1 and 1. This has also made the model stationary.

3.1 Forecasting with the best hybrid models

After fitting different hybrid ARIMA models, two models were selected as the best, which are the hybrid ARIMA-ARCH (2, 1, 1; 3) and the hybrid ARIMA-GARCH (1, 1, 1; 1). These models were then used in the forecasting. The twelve-step out-sample forecast was conducted based on the exchange rate data. The forecast is displayed in figures 4 and 5. The forecast was obtained by using data from previous periods to estimate the exchange rate changes for future occurrences.

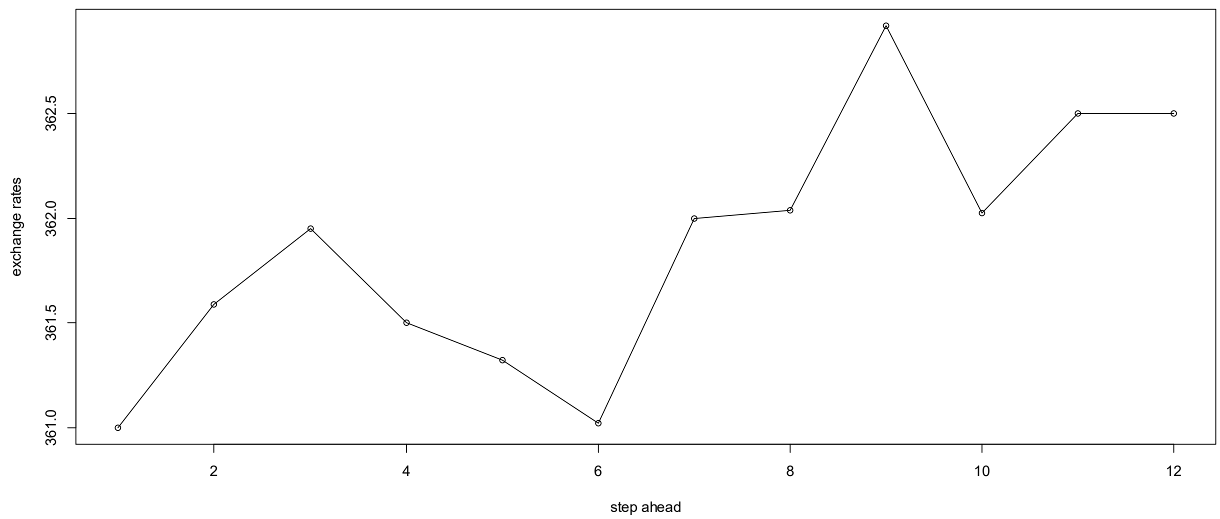

Fig. 4: Forecasting the value of the bureau-de-change exchange rate using the hybrid ARIMA-ARCH (2, 1, 1; 3).

It can be seen in Fig. 4 that the exchange rate increases from January to March 2019 and later decreases to June, which then increases to July and September, decreases in October, and then a slight increase in November and stays steady throughout the year. It seems that hybrid ARIMA-ARCH (2, 1, 1; 3) does a very good job of capturing the dynamic nature of the data and forecasting.

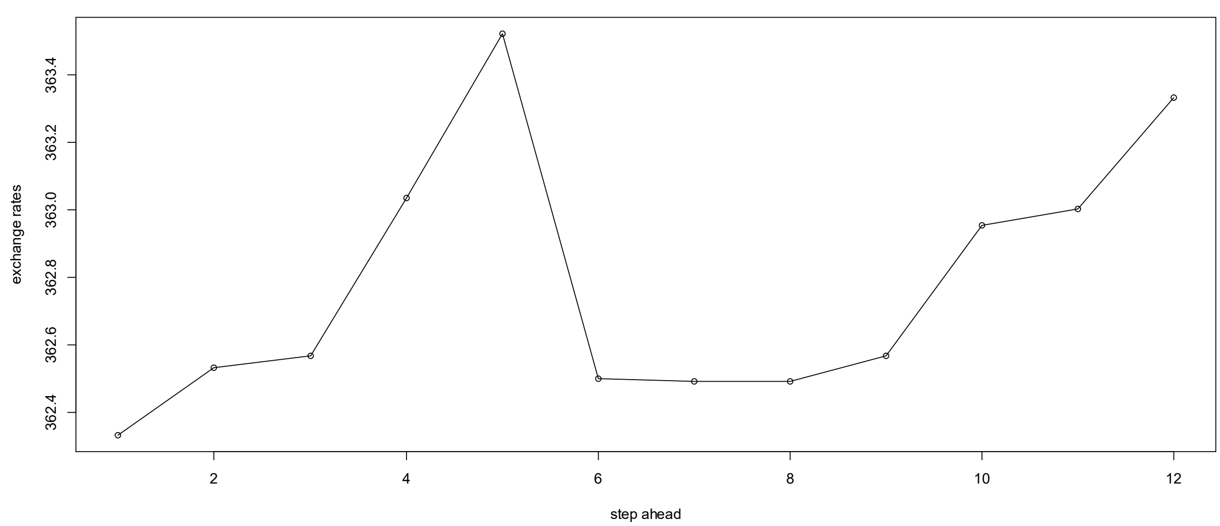

Fig. 5: Forecasting value of bureau-de-change exchange rate using hybrid ARIMA-GARCH (1, 1, 1; 1, 1)

It can be seen in Fig. 5 that the exchange rate slightly increases from January to February 2019 and later increases to May, which then decreases to June, remains steady up to August, and then keeps increasing throughout the year. It seems that hybrid ARIMA-GARCH (1, 1, 1, 1) does a very good job of capturing the dynamic nature of the data and forecasting.

3.3 Comparative performances of the models

Table 5: Best fitted models on bureau-de-change exchange rate data

Model	AIC	BIC
ARIMA(2,1,1)ARCH(3)	5.7597	5.8843
ARIMA(1,1,1)GARCH(1,1)	5.8074	5.8964

From Table 5 are the two best models fitted on the bureau-de-change Exchange Rates data throughout this research work. It can also be seen that, hybrid ARIMA-ARCH model performed slightly better on Bureau-de-change data.

4.0 Conclusion

In this paper, modeling the volatility of Nigeria's foreign exchange (bureau de change) with the proposed hybrid ARIMA-ARCH and hybrid ARIMA-GARCH models was carried out. In terms of the comparative performances of the models, the hybrid ARIMA-ARCH model performed slightly better than the hybrid ARIMA-GARCH model. It can be concluded that the hybrid ARIMA-ARCH captures the volatility of the data better than the hybrid ARIMA-GARCH model. Therefore, these two models are recommended over the traditional Box-Jenkins ARIMA.

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