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Fractal Price Hypothesis and Stock Market Behaviour in Nigeria, Egypt, and South Africa

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Abstract: *The study examined the tangibility and implication of fractal price hypothesis on the behaviour of the Nigerian, Egyptian, and the South African Stock Markets over the study period of January 1995 to December 2020. The study employed secondary data which were gotten from the various sources such as the Central Bank of Nigeria statistical bulletin, World Bank report, and the annual reports of Federal Reserve Economic Data. The employed data analysis techniques are descriptive statistics, the graphical trend of data, univariate data analysis such as the Correlograms Q-Statistics, the Breusch-Godfrey LM tests, autocorrelation test, stationarity test, and the Generalized Autoregressive Conditional Heteroskedasticity. The study observed significant fractal trends in the various countries. In terms of the relationship between fractal prices and liquidity/volume of transactions, the study observed a significant relationship between stock prices and liquidity position of the various countries. While no significant link between fractal prices and returns made by investors was uncovered by the study. Based on the findings of the study, the study concludes that there are fractal price trends in the Nigerian, Egyptian and South African markets and this price trend are observed to affect liquidity and volume of transactions but do not affect investors returns. In the light of the observed findings, the study recommended that; the breadth and width of the Nigerian and Egyptian capital market should be broadened. Also, the extent to which investors insignificantly earned from their trading shows that investors would be prudent to have some of their funds invested in both the stock market and government securities to hedge against potential losses due to exogenous crisis events. The study has added to the growing body of knowledge in the finance and economic debate by considering the peculiar subsect of the weak form hypothesis known as the fractal price hypothesis.*

Keywords: *Fractal Market Hypothesis, Stock Market Behaviour, Efficient Market Hypothesis, Technicalist Analysis*

1. INTRODUCTION

Stock market is an important and active part of nowadays financial market. Both investors and speculators in the market would like to make better profits by analyzing market information (Li, Nishimura, & Men, 2014). So one of the important issues in the stock market is predicted stock price of listed companies and their estimated real value, because prices are signal to guide the effective allocation of capital and liquidity that could be considered as a powerful tool in the efficient allocation of resources (Rounaghi & NassirZadeh, 2016). Therefore, the forecasting of stock price and return has attracted researchers' attention for several decades (Campbell &Yogo, 2006; Kang, Liu, & Qi, 2010). By considering the returns as a time series, most of the forecasting methods utilize statistical models to reveal the hidden properties in the sequence, such as dependency and data distribution (Constantinou, Georgiades, Kazandjian, & Kouretas, 2006; Drakos, Kouretas, & Zarangas, 2010; Hung, 2009).

Financial markets are described as complex systems and the emergence of complexity could be spontaneous and the market crash might happen suddenly with no big changes of economy fundamental parameters. Efficient market hypothesis (EMH) asserts that the financial market is ''informationally efficient''. Over-reaction or under-reaction happens all the time and these noise traders must exist in the market to provide the necessary liquidity for the rational investors (Constantinou *et al.,* 2006). We have seen that empirical evidence shows that the capital markets are not well-described by the normal distribution and random walk theory (Li*et al.,* 2014). The market fails at some time and the crash could happen suddenly with no shifts from economic fundamentals. Market price changes and volatility can be caused by the herding behavior which could be well-described by the complex system.

To explain the above phenomena and the complexity of the financial market, the fractal market hypothesis (FMH) has many conceptual and quantitative advantages over the EMH through modeling and analyzing data. In FMH, financial markets are described as complex dynamical systems that the emergence of complexity could be spontaneous (Matías & Reboredo, 2012; Sonney, 2009). Benoît Mandelbrot's work on fractals showed that much complexity in financial market could be described by certain ubiquitous mathematical laws. Peters proposed FMH first (1991), based on Mandelbrot's work (1982) and using the fractal objects whose disparity parts are self-similar. FMH provides a new framework for the more precise modeling of the turbulence, discontinuity and non-periodicity that truly characterizes today's capital markets. FMH provides a new visual angle of observation to explain the source of market liquidity. The market is stable when it consists of investors covering a large number of investment horizons which ensure that there is ample liquidity for the traders.

Despite the prevailing number of researchers based on the test of market efficiency using the the fractal market hypothesis (FMH) and standard efficient market hypothesis (EMH), an observable trend is that, a majority of these studies are carried out mainly on the more developed markets (Fama & Blume, 1966) and markets of similar capitalization like European and Swedish Stock market (Jennergreen, 1975; Huang, 2005), while other less economically developed markets like Nigeria are less researched. Also, fractal trends have been hypothesized by several research such as; Samuel and Yacout (1981), Olowe (1999), Akpan (1995), and Appiah-Kusi and Menya (2003) who found evidence of weak-form efficiency, but various attempts at determine empirically its existence has been lacking significantly and scanty. The dearth of the empirical literature in Nigeria is not healthy for its aspiration to become one of the top largest economies in the world since policies that seek to attract foreign portfolio investment should be informed by some empirical evidence on the stock market efficiency (Suzuki & Yasuda, 2006).

Past studies that undertook the fractal market hypothesis test such as (Alam, 2017; Alade, Adeusi, & Alade, 2020; Samson, Onwukwe, & Enang, 2020; Osabuohien, 2020) have been observed to use various tools and techniques which are far from the main tools such as the box count method etc.. Also, Nigeria, Egypt, and South Africa are the largest economies in Africa and seek to be among the twenty largest economies in the world in the year 2022. The behaviour of the stock market in this regard cannot be over-emphasized, for the long-term fund is a critical factor in the economic transformation process. More so, stock markets afford investors the opportunity to diversify their portfolios across a variety of assets. Given this importance of the stock market, it is imperative to test whether the behaviour of the stocks in the Nigerian, Egyptian, and the South African Stock Markets follow the efficient or the fractal market hypothesis. The basic question this study set out to investigate is: Does the Nigerian, Egyptian, and the South African Stock Market conform to Fractal behaviour? So, testing the absolute fractal reality of these markets seems to be the most informative method of gauging the efficiency of the markets (Alam, 2017). The relative efficiency, that is the efficiency of one market or market index, measured against the other, appears to be another useful concept in traditional finance literature. It is therefore, imperative to investigate the fractal market hypothesis, which can help identify the nature of efficiency or inefficiencies of the Nigerian, Egyptian, and the South African Stock markets.

The aim of this study is to examine the existence and implication of fractal price hypothesis on the behaviour of the Nigerian, Egyptian, and the South African Stock Markets. The specific Objectives are to:

i. Examine the fractal autocorrelation stock prices in the Nigeria, Egyptian, and South African Stock Markets.
ii. Analyze the nature of sensitivity of stock prices to market liquidity in the Nigerian, Egyptian, and the South African Stock Market; and
iii. investigate the profit/return implication of fractal stock price on the Nigeria, Egyptian, and South African stock market returns.

Research Questions

In light of the aforementioned objectives, the study asks the following question.

i. What is the nature of fractal autocorrelation in the stock prices of Nigeria, Egypt, and South Africa Stock Market?
ii. What is the nature of sensitivity between stock prices and market liquidity in the Nigerian, Egyptian, and the South African Stock Market?
iii. How does the fractal stock prices affect the rate of returns of the Nigerian, Egyptian, and the South African Stock Market?

In view of the proposed objectives and research questions, the study states the following null hypotheses (**$H_{0:}$**) as follows;

H_{01}: No significant fractal autocorrelation can be observed in the Nigerian, Egyptian, and the South African Stock Market prices.

H_{02}: Stock prices are not significantly sensitive to market liquidity in the Nigerian, Egyptian, and the South African Stock Market.

H_{03}: Fractal stock price do not significantly affect the rate of return of stock prices in Nigeria, Egyptia, and South Africa's stock market.

The study covers the application of the fractal market hypothesis in the Nigerian stock market and Egyptian stock market and the South African stock market towards evaluating the efficiency of the market. There are 29 exchanges in Africa, representing 38 nations' capital markets. The study uses monthly All Shares Index, Volume of Transaction, and returns in Nigeria Stock Market, Egyptian Stock Market, and the South African Stock market over a cross-sectional period of 1st of January 1995 to 31st December 31, 2020. The choice of the study period is informed by the end of the South African Apartheid era which caused massive disruption in the economic operation of the country and would affect the results of the study if included. The choice of the countries (Nigeria, Egypt, and South Africa) is informed by the number of listing/listed firms which are all above 200. larger sample sizes (in this case being the aggregated listing) give more reliable results with greater precision and power (Krejcie & Morgan, 1970). The study was limited to the listed companies in Nigerian, Egyptian, and the South African Stock Exchange. The study dwells on the organizational/institutional unit of analysis.

While this section introduces the study. Section two reviews the related literature of the study which includes the Theoretical, Conceptual, and Empirical Framework. And end with the highlight of the relevant knowledge gap to be filled by the study. Section three examines the method of study; it comprises of the research design, sources of data collection, and discusses the procedures used to obtain the data, the reason for using this method, reliability and validity of the study, population and sampling procedure, and technique model specification and the method of data analysis of the regression results. Section four entails the presentation of data, which employs a quantitative approach to analyzing the employed data for the study, which entails multivariate analysis. While in section five, ends the research work with the summary of findings, conclusions showing the heuristic knowledge derived from the study and policy recommendations and areas for further research. It discusses managerial implications and provides the conclusion, recommendation, and contribution to knowledge in light of the study.

2. LITERATURE REVIEW

2.1. Conceptual Framework

2.1.1 The Concept of the Fractal Market Hypothesis

Primarily, fractals are infinitely complex patterns that are self-similar across different scales. They are created by repeating a simple process over and over in an ongoing feedback loop (Karp & Van Vuuren, 2019). Therefore, the fractal markets hypothesis (FMH) dictates that financial markets, and particularly the stock market, follow a cyclical and repeatable pattern. It is an alternative investment theory to the widely utilized efficient market hypothesis (EMH). It analyzes the daily randomness of the market and the turbulence witnessed during crashes and crises (Moradi, Jabbari Nooghabi, & Rounaghi, 2019; Liu, Zhou, Zhu, & Wang, 2020). Fractal markets hypothesis analyzes the daily randomness of the market—a glaring absence in the widely utilized efficient market hypothesis. It examines investor horizons, the role of liquidity, and the impact of information through a full business cycle. The market is considered stable when it is comprised of investors of different investment horizons given the same information. Overall, Fractal market hypothesis seeks to explain investor behaviors in all market conditions, something the popular efficient market hypothesis fails to do. The fractal markets hypothesis (FMH) dictates that financial markets, and particularly the stock market, follow a cyclical and repeatable pattern. One thing it has in common with EMH is that both theories rely heavily on the prevalence of information with investors (Lamphiere, Blackledge, & Kearney, 2021). From there, they take different paths. According to the fractal markets hypothesis (FMH), during stable economic times, information does not dictate investment horizons and market prices.

2.2 Theoretical Framework

The base line theories supporting and opposing the fractal market hypothesis are presented as follows;

2.2.1 Chaos Theory

Chaos theory is a controversial and complicated theory that has been used to explain some features of systems that have traditionally been difficult to accurately model. This suggests that periods of low-price volatility do not necessarily reflect the true health of the market. The theory was summarized by Edward Lorenz (1963) as: Chaos: When the present determines the future, but the approximate present does not approximately determine the future. Chaotic behavior exists in many natural systems.

Falling into the framework of chaos theory, the fractal markets hypothesis (FMH) explains markets using the concept of fractals-fragmented geometric shapes that can be broken down into parts that replicate the shape of the whole. With respect to markets, one can see that stock prices move in fractals. Due to this characteristic, technical analysis is possible: in the same way that the patterns of fractals repeat themselves along all time frames, stock prices also appear to move in replicating geometric patterns through time. That analysis focuses on the price movements of assets based on the belief that history repeats itself. Following this framework, the fractal markets hypothesis (FMH) studies investor horizons, the role of liquidity, and the impact of information through a full business cycle (Maksyshko & Vasylieva, 2019; Aleksandrovna, 2019).

2.2.2 The Fractal Market Hypothesis

A fractal approach is used to analyse financial time series by applying different degrees of time resolutions. This leads to the heterogenous market hypothesis, where different market participants analyse past events and news with different time horizons. A new general model for asset returns is studied in the framework of the FMH (Weron & Weron, 2000). Financial markets are described as complex systems, and the emergence of complexity could be spontaneous, and the market crash might happen suddenly with no big changes of economy fundamental parameters. EMH asserts that the financial market is

"informationally efficient." Overreaction or underreaction happens all the time, and these noise traders must exist in the market to provide the necessary liquidity for the rational investors. We have seen that empirical evidence shows that the capital markets are not well described by the normal distribution and random walk theory (Johnson, Jefferies, & Ming Hui, 2003; Stanley, 1995; Mantegna, 1999; Mantegna & Stanley, 1997). The market fails at some time, and the crash could happen suddenly with no shifts from economic fundamentals. Market price changes and volatility can be caused by the herding behaviour that could be well described by the complex system. To explain the above phenomena and the complexity of the financial market, the FMH has many conceptual and quantitative advantages over the EMH through modelling and analysing data. In FMH, financial markets are described as complex dynamical systems that the emergence of complexity could be spontaneous (Mantegna, Palagyi, &Stanley, 1999; Mantegna & Stanley, 1996; Münnix, Shimada, & Schäfer, 2012). Benoît Mandelbrot's work on fractals showed that much complexity in financial market could be described by certain ubiquitous mathematical laws. Peters proposed FMH first (Edgar & Peters, 1994), based on Mandelbrot's work (Mandelbrot,1982) and using the fractal objects whose disparity parts are self-similar. FMH provides a new framework for the more precise modelling of the turbulence, discontinuity, and non-periodicity that truly characterizes today's capital markets (Li *et al.,* 2014).
FMH takes into account the daily randomness of the market and anomalies such as market crashes and stampedes. It proposes that;

(a) market is stable and has sufficient liquidity when it comprises investors with different time horizons,
(b) these investors stay in their "preferred habitat" (time horizon), no matter what the market information indicates,
(c) the available information may not be reflected in the market prices, and
(d) the market prices trend indicates the changes in expected earnings (which mirror long-term economic trends).

2.2.3. Efficient Market Hypothesis (EMH)

The Efficient Market Hypothesis (EMH) states that in an efficient market, stock prices adjust so quickly to new information that:

(a) Security prices fully reflect all available information.
(b) Successive changes in security prices are in dependent (Okafor, 1983; Fama, 1970).

Defining efficient market in this way has the following important implications:

(1) An investor cannot use available information to earn non-zero abnormal returns.
(2) In an efficient market, when new information is added to the information set, its revaluation implications are unbiased and instantaneously impounded into the current market prices (Rahman & Hossain, 2006).
(3) The real financial position of a company will in the long run always be reflected in company's share price. If management makes a positive investment decision, and is made known to the public security price will always reflect the manager's action (Ibenta, 2005).
(4) In a situation where the market is efficient, investor's choice is passive portfolio strategy, but in a not so efficient market, the key element to investors' choice is active portfolio management to enable them detect and exploit perceived departures from efficiency (Bodie *et al.,* 2007).

The Weak Form Hypothesis

The weak-form hypothesis posits that stock prices already reflect all information that can be derived by examining market trading data such as the history of past prices, trading volume or short interest (Bodie *et al.,* 2007). Weak-form efficiency also means that unanticipated return is not correlated with previous unanticipated returns. In other words, the market has no memory; knowing the past does not help to earn future returns. This version of EMH implies that trend analysis is fruitless. Past stock price data are publicly available and virtually costless to obtain. This version holds that if such data ever conveyed reliable signals about future performance, all investors would have learned already to exploit the signals. Ultimately, the signals lose their value as they become widely known because a buy signal, for instance, would result in an immediate price increase. In a weak-form efficient market, past prices and volume data are already impounded in security prices and no amount of chart reading or any other trading device is likely to consistently outperform the buy and hold strategy.

2.3 Empirical Review

Lamphiere, Blackledge, and Kearney (2021) examined the trend prediction results based on back testing of the European Union Emissions Trading Scheme futures market. This is based on the Intercontinental Exchange from 2005 to 2019. An alternative trend prediction strategy is taken that is predicated on an application of the Fractal Market Hypothesis (FMH) in order to develop an indicator that is predictive of short-term future behaviour. To achieve this, we consider that a change in the polarity of the Lyapunov-to-Volatility Ratio precedes an associated change in the trend of the European Union Allowances (EUAs) price signal. The application of the FMH in this case is demonstrated to provide a useful tool in order to assess the likelihood of the market becoming bear or bull dominant, thereby helping to inform carbon trading investment decisions. Under specific conditions, Evolutionary Computing methods are utilised in order to optimise specific trading execution points within a trend and improve the potential profitability of trading returns.
Ikechukwu,Evans and Olaniyi (2020) this study investigated the weak axiom of the efficient market hypothesis (EMH) as it applies to fifteen (15) leading stock markets in Africa. There are currently over twenty-nine stock exchanges in Africa with a significant degree of disparities ranging from market size, trading volume, number of listed companies, access to funds, access to information to market standardization etc. The article deviated from the conventional linear approach of testing efficient market hypothesis and the method of using the runs test for serial dependency to test the weak-form efficient market hypothesis. The study adopted the wavelet unit root analysis-tool, which decomposed the stochastic processes into its wavelet components, with varying frequency band. The study found that institutional constraints have implications for the efficient market hypothesis and investment in the African stock market. The conclusions drawn from the study is the relevance of using past historical stock prices to predict the current earnings at stock markets in Africa, a negation of the efficient market hypothesis.

Robert Amoah (2020) conducted study on meta-analysis of existing research reports involving the efficient market hypothesis theory in both emerging and frontier markets to establish the efficacy of the theory in those markets. Findings of this research hopefully would provide invaluable contribution to the body of finance literature, and to offer intriguing insights to finance students, professors, and academicians as to how and why the theory works in developed markets but fails in most emerging and frontier markets. Small market size and liquidity, lack of awareness of financial markets' products and activities, lack of availability of financial education programs, lack of efficient operational framework, and lack of competitive landscape were found as major factors which explain the high degree of weak-form of market efficiency as characterized by emerging, frontier, and standalone markets.

In this study of Liu, Zhou, Zhu, and Wang (2020), variance ratio test and detrended fluctuation analysis are adopted to measure the efficiencies of Beijing, Guangdong, Hubei, Shanghai, Shenzhen, Tianjin and Chongqing seven carbon markets in China. The empirical results indicate that on the basis of efficient market hypothesis, adopts variance ratio test, Hubei and Guangdong markets are weak efficient, Beijing, Shenzhen and Shanghai markets are inefficient, and the efficiencies of Tianjin and Chongqing are uncertain under daily returns. All seven carbon markets are weak efficient under weekly returns. On the basis of fractal market hypothesis, Hurst exponents are obtained by detrended fluctuation analysis, Shanghai, Guangdong and Hubei markets are weak efficient, the other four markets are inefficient under daily returns. Beijing and Hubei markets are weak efficient and the other five markets are inefficient under weekly returns. On the whole, the efficiencies of China's carbon markets are generally low. Combining the empirical results with the reality of China's carbon markets, it's proved that the results of fractal market hypothesis are more convincing. In comparison to efficient market hypothesis, fractal market hypothesis can more effectively measure the efficiency of carbon market.

The rate at which financial crises repeat themselves over time in similar and recurring 'greed-panic', 'boom-bust' patterns throughout history irrespective of technological advancements are contrary to those suggested by classical finance theories. For example, as pointed out by Mallaby (2010: 105):"...a plunge of the size that befell the S&P 500 futures contracts on October 19 [1987] had a probability of one in 10160—that is, a'1' with 160 zeroes after it. To put that probability into perspective, it meant that an event such as the crash would not be anticipated to occur even if the stock market were to remain open for twenty billion years, the upper end of the expected duration of the universe, or even if it were to be reopened for further sessions of twenty billion years following each of twenty successive big bangs.

Mai Ahmed Abdelzaher (2020) the study examined is if the Egyptian stock market has information efficiency (market efficiency assumptions) by studying the presence of time series properties for daily stock returns between 2005 and 2015. Parametric and non-parametric tests are used to achieve this purpose, such as ADF/ PP unit root- RUNS TEST-Perron - run test. The Jarque– Bera test was used to measure the moderation of returns; the GARCH model and ARCH model are used also. The results referring to the Egyptian stock market follow the inefficient form, and the prices are closer to random traffic standards, showing that the price changes are random. Thus, there may be shares presented at less than their real value. Additionally, the consequence of the inefficiency of the Egyptian stock market on the weak level, given that the prices of stock prices do not reflect all historical information, it is possible for market participants to achieve unusual returns by using historical prices of shares.

”Velasquéz (2009) proposes adapting Chaos Theory and Fractal science to explain financial phenomenon as this new theory would be able to explain the “messiness” of financial markets of today with the implication that the underlying assumptions of several classical finance theories such as the rationality of market participants, efficient markets and models of equilibrium have to be discarded.

The shortcomings of Classical Finance Financial crises, such as the ones that occurred in 1987, 1998, 2000 and then recently in 2007 and the corona effect of 2019/2020, have been brushed off as anomalies by proponents of the Efficient Market Hypothesis (EMH) who maintain that markets remain informationally efficient. However, the frequency with which these crises occur cannot be explained by the underlying assumptions of an efficient market. Mandelbrot (2012) proves that the substantial one-day market fluctuations cannot be brushed aside as anomalies when estimating risk or forecasting returns since removing the ten largest one-day fluctuations from the S&P 500 over a period of 20 years, gives a very different kind of market reality and therefore these big moves do matter.

Ghazani and Ebrahimi (2019) examines the existence of the fractal market hypothesis (FMH) as an evolutionary alternative to the efficient market hypothesis (EMH) by applying daily returns on the three benchmark crude oils. The data coverage of daily returns is from 2003 to 2018. The automatic portmanteau and generalized spectral tests are applied in this study. The results show that the Brent and the WTI oil markets possess the highest efficiency levels. In addition, the behavior of OPEC basket data represents that when we approaching toward longer window lengths (e.g. from 100 to 500-days); the degree of conformity with AMH decreases.

Moradi, Jabbari Nooghabi, and Rounaghi (2019) applies L-Co-R coevolutionary algorithm for forecasting and analysis of timeseries stock returns in an attempt to observe the fractal trend of the sampled study markets. The study examined the daily, monthly, and yearly time series stock returns on Tehran Stock Exchange and London Stock Exchange over a period from 2007 to 2013. The statistical analysis in London Stock Exchange shows that the L-Co-R algorithm outperforms to the other rmethods, regardless of the horizon, and is capable of predicting short, medium, or long horizons using real known values. The statistical analysis in Tehran Stock Exchange shows that the L-Co-R algorithm outperforms to the other methods and is capable of predicting only short and medium terms. Thus, fractal market hypothesis was accepted for Tehran Stock Exchange and rejected for London Stock Exchange.

Karp and Van Vuuren (2019) in an attempt to understand the fractal evidence in a capital market evaluated the time dependence of the supply of investors’ liquidity to the market is explored for two developed market indices and one emerging market index. The study employed a quantity known as the Hurst exponent determines whether a fractal time series evolves by random walk, a persistent trend or mean reverts. Another quantity, the fractal dimension of a time series, provides an indicator for the onset of chaos when market participants behave in the same way and breach a given threshold. A relationship is found between these quantities: the larger the change in the fractal dimension before breaching, the larger the rally in the price index after the breach. In addition, breaches are found to occur principally during times when the market is trending.

Kumar, Jayakumar, and Kamaiah (2017) an attempt is made to test the Fractal Markets Hypothesis (FMH) which states that a financial market can plunge into crisis when a particular trading time horizon gains prominence over others applied a wavelet-based method to capture the activities in different timescales. The study tested the proposition for nine Asian forex markets of China, India, Hongkong, Japan, South Korea, Singapore, Sri Lanka, Taiwan, Thailand for the period from 05-01-1994 to 30-06-2017. The study used the bilateral daily exchange rate of the corresponding currencies against the U.S Dollar. The time period covers two major crises and they are the 1997-8 East Asian currency crisis and the 2008 global financial crisis. The study captured both the events and from the wavelet spectra it is evident that the crisis period distinguishes itself with increased activity by the short-term traders as proposed by the FMH. The study found that the 1997-8 crisis affected not only the East Asian markets but also the other forex markets as well.

Prince Kwasi Sarpong (2017) applied the Fractal Market Hypothesis (FMH) formalised within the framework of Chaos Theory, to explain the existence of the low volatility anomaly on the JSE. Building upon the Fractal Market Hypothesis to provide evidence on the behaviour of returns time series of selected indices of the JSE, the BDS test is applied to test for non-random chaotic dynamics and further applies the rescaled range analysis to ascertain mean reversion, persistence or randomness on the JSE. The BDS test confirms that all the indices considered in this study are not independent and identically distributed. Applying the re-scaled range analysis, the FTSE/JSE Top 40 and the FTSE/JSE All Share Index appear relatively efficient and riskier than the FTSE/JSE Small Cap Index, which exhibits significant persistence and appears to be less risky and less efficient contrary to the popular assertion that small cap indices are riskier than large cap indices. The study further analyses the three fundamentals of the FMH namely, the impact of information, the role of liquidity and time horizon on the top 40 and small cap indices. Information is not uniformly distributed among the two indices as the FTSE/JSE Top 40 index receives more publications form sources such as newspapers, online publications and journals as well as JSE issued news and historical company new.

It is evident from this review of literature that there are two schools of thought about weak-form efficient market Hypothesis. On the one hand, one of them argues that markets are efficient and returns are unpredictable. The works of Kendal (1953), Fama (1965), Dickinson and Muragu (1994), Ojah and Karemera (1999), are some of the works which largely conclude that the stock market is efficient. On the other hand, the works of Banz (1981), Arbel (1985), Poterba and Summers (1988), Lo and Mackinlay (1988), etc, document empirical evidence of anomalies that appeared to contradict the theory of efficient markets. Among other findings, stocks returns are found to be negative from the close of trading on Friday to the close of trading on Monday (Day of the week effect). The average annual returns of small firms are greater than the average returns of large firms (small firm effect) and the small firm effect occurs virtually entirely in January (January effect). Moreover, poorly performing stocks in one period experience sizeable reversal over the subsequent period, while the best-performing stocks in a given period tend to follow with poor performance in the following period (Reversal effect) and a powerful predictor of returns across securities is the ratio of the book value of firm's equity to the market value of equity (Book to market effect).

From this review, also, it is clear that the empirical literature on the weak-form efficiency of the NSE and the Egyptian Stock Exchange are few and these few existing kinds of literature produced mixed evidence. While some researchers provide evidence showing that the NSE is weak-form efficient, others debunked such evidence. The researchers who found evidence of weak-form efficiency on the NSE are Samuel and Yacout (1981), Ayadi (1984), Anyanwu (1998) and Olowe (1999); Whereas Akpan (1995) and Appiah-Kusi and Menya (2003) found the NSE weak-form inefficient. While these studies on the Nigeria Stock Exchange offered evidence by analyzing the price series of samples of individual stocks, none of them used a long-time period which, according to Rahman and Hossain (2006), reduces the problem of infrequent trading bias. Also, these studies use parametric tests without hypothesizing the distribution of market returns for normality, which is one of the conditions for using parametric methods. Furthermore, none of these studies compared weak-form efficiency across time for the NSE and Egyptian Stock Exchange.

This study seeks to fill these lacunas by empirically examining the fractal nature of the weak-form efficiency evidence on the Nigerian, Egyptian, and the South African Stock Exchange. In doing so, it will make a significant contribution to the extant literature. Firstly, it will provide empirical evidence using the monthly All-Share Index of the Nigerian, Egyptian, and the South African Stock Exchange and we will also use a longer–time period. These will overcome the weakness associated with infrequent trading of some of the individual stocks. Secondly, it will examine the distribution of the excess stock returns for normality and a non–parametric test will be used to analyze data so that non–normal distribution will not bias findings. Thirdly, unlike prior studies which test absolute efficiency of the Nigerian, Egyptian, and the South African Stock Exchange, it will compare efficiency across time for the both countries. Finally, this study will extend the existing evidence by using recently available data. Thus, this study will not only overcome the weaknesses of the earlier studies but will also establish the true position of the Nigeria Stock Exchange and Egyptian Stock Exchange with respect to the level of its fractal weak-form efficiency.

Many studies had been conducted on various aspects of trading in the Nigerian Stock Exchange, but none had focused on the profitability and applicability of filter rule test especially as related to the Nigerian stock market and Egyptian stock market. This study is therefore significant in that it offered an in-depth analysis of the applicability of the fractal efficient hypothesis in the Nigerian, Egyptian, and the South African stock Exchange. This literature will go a long way in helping determine the fractal trends of the Nigerian, Egyptian, and the South African stock market.

3. METHODOLOGY

This section focuses on the methodological framework adopted to investigate the dynamics of Fractal Market Hypothesis in Nigeria, Egypt, and South Africa. Thus, the researcher introduces econometric equations to capture the objectives of this study. In this regard, the section is structured in different subsections: research design, population of the study and sample size, data, model specification, estimation technique.

Research Design: The study adopted the *ex-post facto-hypothetico deductive* research design, since it employed historical data in conjunction with quasi-experimental based research design. The study also used the descriptive research design derived from actual observation of the stock market indices exchanges on daily basis to investigate the characteristics of the fractal market trends.

Population for the study: The study's target populations are the prices of stocks of all the companies listed on the floor of The Nigerian, Egyptian, and the South African Stock Exchange. This is expected to reflect the activities taking place in these stock exchanges so as to enhance the testability of the Fractal Market Hypothesis. As such, the study population is based on all the companies that are listed in Nigerian, Egyptian, and the South African Stock Exchanges. The NSE commenced official operation in 1961 with 19 listed securities and 4 dealers. As of November 2019, it has a total of 161 listed companies, with 8 domestic companies on the premium board, 144 companies on the main board, and 4 on the Alternative Securities Market (ASeM) board. The market has equity capitalization of N20,668,961,969.85 and all share index of 39,504.50. It is the largest stock exchange in the Sub-Saharan African countries, but relatively less liquid and characterized with thin trading.

Sample and Sampling technique: The study employed the purposive/non-probability sampling technique based on market listing of the various stock markets and the availability of data. The market indices are used for the three markets. A non-random sample technique was adopted in this study for the selection of countries whose data make up met the purpose of this study in term of consistency in trading and market capitalization. Therefore, countries that did not have sufficient listing over the proposed sample period were excluded from the sample group. In view of this, three highly capitalized stock markets with sufficient listing sizes were selected, namely Nigerian, Egyptian, and the South African Stock Exchanges. Thus, our sample size comprises these three stock exchanges in Africa using there All Share Index, volume of transaction, and average rate of return. To confirm the degree of prediction using the Fractal, the overall market is also included to the sample size. This explicitly implies that we have data for the all the companies embedded in the market indices.

Nature/Sources of Data: The study employed daily stock market indices of the selected market Nigerian, Egyptian, and the South African stock markets respectively for a period ranging from January 1st , 1995 to December 30th ,2020 to cover part of the periods of COVID-19 Pandemic**.** The data are collectively sourced from a website-nginvesting.com. Secondary data will be used in this research. The use of daily market indices for this study is in line with some of the previous empirical studies on weak form efficiency on emerging stock markets around the world (Lamphiere, Blackledge, & Kearney, 2021; Urrutia, 1995; Olowe, 1999; Simon & Laryea, 2004; Branes, 1986; Claessens *et al.,* 1995; Rahman & Hossain, 2006).

Researchers have looked at the concept of market efficiency in various ways and analyzed it using different models. The hypothesis has been tested using different statistical techniques and in different markets over different time periods. The volume of research in this area has led to numerous advances in both theoretical modeling and statistical analysis surrounding the EMH. However, despite all these advances it still appears that the EMH is not yet empirically well-defined and the **FMH** therefore remains a critical attempt at defining the EMH. The empirically testable model of FMH includes too many assumptions such that the ultimate test for FMH have becomes a test for several ancillary hypothesis as well (Lo, 1997). This section is therefore concerned with the methods and procedure to be employed in testing the applicability of fractal market trends in the Nigerian, Egyptian, and the South African stock exchange.

Operational Measures of Variables

The fractal price hypothesis is measured with stock prices as captured using the all share prices, while the stock market behaviour is measured using both the liquidity and return rate as stipulated by the fractal price hypothesis (Zhao, Zhu, & Xia, 2016).

Liquidity: This is measured using the volume of transaction. The volume of transaction according to Bodie, Kane, and Marcus (2007) reflects the ease at which a security can be bought or sold in a secondary market which is replicative of the stock market behaviour. In the investing world, the terms "bull" and "bear" are frequently used to refer to market conditions. These terms describe how stock markets are doing in general—that is, whether they are appreciating or depreciating in value. Because the financial markets are greatly influenced by investors' attitudes, these terms also denote how investors feel about the market and the ensuing economic trends. Since it is hard to time a market bottom, investors may withdraw their money from a bear market and sit on cash until the trend reverses, further sending prices lower.

Stock Prices: This is measured using the All-Share Index of Nigeria, Egypt, and South Africa.

Return Rate: This is measured using the changes in the annual values of the All-Share Index of the various sample countries (Nigeria, Egypt, and South Africa).

Model Specification

Toeing the line of the studies by David, Inacio Jr, Nunes, and Machado (2021), Frezza (2018), and Wang, Li, and Shen (2019) the study presents its models in the following auto-regressive form as;

$$STK_t = f(STK_{t-1\ldots}STK_{t-x}) \quad 3.1$$

$$LQD_t = f(STK_t, LQD_{t-1\ldots}STK_{t-x}, LQD_{t-x}) \quad 3.2$$

$$RRT_t = f(STK_t, RRT_{t-1\ldots}\ STK_{t-x}, RRT_{t-x}) \quad 3.3$$

Converting to econometric form by the introduction of the constant term and error term (μ_t, ψ_t):

$$STK_t = \text{ϡ}_0 + \text{ϡ}_1 STK_{t-1} + \ldots + \text{ϡ}_1 STK_{t-x} + \Omega_t \quad 3.4$$

$LQD_t = \alpha_0 + \alpha_1 STK_t + \alpha_2 LQD_{t-1} + \ldots + \alpha_3 STK_{t-x} + \alpha_4 LQD_{t-x} + \mu_t$ 3.5

$RRT_t = \beta_0 + \beta_1 STK_t + \beta_2 RRT_{t-1} + \ldots + \beta_3 STK_{t-x} + \beta_4 RRT_{t-x} + \ddot{Y}_t$ 3.6

Where:

STK = Stock prices

LQD = Liquidity

RRT = Rate of Return of Stock Prices

$\alpha/\beta/ઝ_0$ = Constant variable/Intercept

$\alpha/\beta/ઝ_{1-3}$ = Slope/Coefficient

$\mu/\ddot{Y}/\Omega_t$ = Error Terms/Stochastic variables

$_t$ = time series

$_{t-x}$ = Allowable lag of variable (Based on lag selection criteria)

Apriori Expectations:

The standard expectation for the study is an efficient market void of fractal trends, therefore, based on theories and empirical studies, the predictor variables are expected to display missed and insignificant results on the criterion variable. This is as a result of the fact that an increase in the stock market behavior should shows weak relationship with fractal prices and trends.

Methods of Data Analysis

To understand the nature and type of relationship between employed variables, the study employs the Stepwise Autoregression evaluation, Long Run Bounds Test, and Causality Test.

Graphical Presentations and Descriptive Statistics: To test for the visual presence of fractal trend, the study employed the use of graphs, descriptive statistics, autocorrelation test, and runs test. Descriptive statistics are very important because if the study simply presented the study raw data it would be hard to visualize what the data was showing, especially if there was a lot of it. Descriptive statistics, therefore, enables us to present the data in a more meaningful way, which allows simpler interpretation of the data.

Box Counting Method and Dynamic Time Warping Method for Fractal Trends: The study employed the box counting method. Box counting is a method of gathering data for analyzing complex patterns by breaking a dataset, object, image, etc. into smaller and smaller pieces, typically "box"-shaped, and analyzing the pieces at each smaller scale. In fractal geometry, the Minkowski–Bouligand dimension, also known as Minkowski dimension or box-counting dimension, is a way of determining the fractal dimension of a set S in a Euclidean space R^n, or more generally in a metric space.

Dynamic Time Warping (DTW) on the other hand is a way to compare two -usually temporal- sequences that do not sync up perfectly. It is a method to calculate the optimal matching between two sequences.

Serial Correlation Test: To undertake the presence of serial or autocorrelation, The Breusch–Godfrey test is a test for autocorrelation in the errors in a regression model. The null hypothesis is that there is no serial correlation of any order up to p. Because the test is based on the idea of Lagrange multiplier testing, it is sometimes referred to as an LM test for serial correlation, while the correlogram is a commonly used tool for checking randomness in a data set. If random, autocorrelations should be near zero for any and all time-lag separations. If non-random, then one or more of the autocorrelations will be significantly non-zero.

Stationarity (Unit Root) Test: It is crucial to examine the stationarity qualities of time series data in order to avoid the problem of spurious estimations. In this sense, the Augmented Dick-Fuller (ADF) test is employed. For decision, the ADF statistics for the respective study variables should on absolute terms, be more than the corresponding Mackinnon critical values at 1%, 5%, and 10% levels of significance for the null hypothesis of non-stationarity to be rejected. Failure to attain stationarity of the variables would provide for subsequent differencing for stationarity to be effected.
Assume that Y_t is random walk process, $Y_t = Y_{t-1} + p._t$, then the regression model becomes $Y_t = pY_{t-1} + p._t$. Subtract Y_{t-1} from both sides of the equation,

Yt-Yt-i = ajYt-j-Yt-i + Ut (5)

AYt = (a-i)Yt-i + Ut (6)

AYt = (a-i)Yt-i+ a2T + Ut (7)

Where a-1= $p_{i\}}$ A is change in Y_t or first difference operator and t is the trend factor. u_t is a white nose residual.

AYt = piYt-i + Ut (8)

With a drift the study have;

AYt = ao + piYt-i + Ut (9)

In practice, the study tests the hypothesis that*p=0. Ifp=0, "a"* in equation 5 will be equal to 1, meaning that the study has a unit root. Therefore, the series under consideration is non-stationary. In the case where *p > 0,* that is, the time series is stationary with zero mean and in the case of equation 6, the series, Y_t is stationary around a deterministic trend. If p >1, it means that the underlying variable will be explosive.

However, conducting the DF test as in (6) or (7), it is assumed that U_t is uncorrelated. But in the case the error terms (Ut) are correlated, the Augmented Dickey-Fuller (ADF) is resorted to, since it adjusts the DF test to take care of possible autocorrelation in the error terms (Ut), by adding the lagged difference term of the dependent variable, AY_t.

Lag Length Selection: Due to the fact that previous credits (especially in the long term) may be influencing current results more than current disbursements. This, therefore, necessitates the inclusion of lag length selection. Estimating the lag length of the autoregressive process for a time series is a crucial econometric exercise in most economic studies. This study attempts to provide helpful guidelines regarding the use of lag length selection criteria in determining the autoregressive lag length. There are several criteria for choosing the optimal lag length in a time-series: AIC: Akaike information criterion; BIC: Schwartz information criterion; HQ: Hannan-Quinn criterion; RMSE: Root Mean Square Error; MAE: Mean Absolute Error; BP: Bias proportion; LIK: Log-Likelihood. The discrimination function differs from one to another criterion. In general, the study uses all criteria cited above and after that, the study takes the smallest lag length from them.

Autoregressive Conditional Heteroskedasticity (ARCH): Autoregressive conditional heteroskedasticity (ARCH) is a statistical model used to analyze volatility in time series in order to forecast future volatility. In the financial world, ARCH modeling is used to estimate risk by providing a model of volatility that more closely resembles real markets. ARCH modeling shows that periods of high volatility are followed by more high volatility and periods of low volatility are followed by more low volatility. In practice, this means that volatility or variance tends to cluster, which is useful to investors when considering the risk of holding an asset over different time periods. The ARCH concept was developed by economist Robert F. Engle III in the 1980s. ARCH immediately improved financial modeling, resulting in Engle winning the 2003 Nobel Memorial Prize in Economic Sciences.

Generalized autoregressive conditional heteroscedasticity (GARCH): This is a Process is an econometric term established in 1982 by Robert F. Engle: It is usually model as follows:

$Var(yt|yt-1,\ldots,yt-m)=\sigma 2t=\alpha 0+\alpha 1y2t-1+\cdots+\alpha my2t-m$.

Decision Rule: y_t is white noise when $0 \leq \alpha_1 \leq 1$.

Impulse Response: refers to "the response of any dynamic system in response to some external change. In both cases, the impulse response describes the reaction of the system as a function of time".

$y_t = y^s + Ay^h_{t-1} + Bu_t$

Variance Decomposition: The variance decomposition "designates the amount of information each variable contributes to the other variables in the autoregression. It determines how much of the forecast error variance of each of the variables can be explained by exogenous shocks to the other variables". It is usually model as follows:

$y_t=v + A_1y_{t-1}+\ldots+A_p y_{t-p}+u_t$

4. RESULTS AND DISCUSSIONS

Data Analysis

The study examined the tangibility and implication of fractal price hypothesis on the behaviour of the Nigerian, Egyptian, and the South African Stock Markets over the study period of January 1995 to December 2020. The study employed secondary data which were gotten from the various sources such as the Central Bank of Nigeria statistical bulletin, World Bank report, and the annual reports of Federal Reserve Economic Data. The employed data analysis techniques are descriptive statistics, the graphical trend of data, univariate data analysis such as the Correlograms Q-Statistics, the Breusch-Godfrey LM tests, autocorrelation test, stationarity test, and the Generalized

Estimation of Fractal Trend Using the Box-Counting Method and Dynamic Time Warping of Runs Test.

Since a fractal can be defined as a self-similarity property, i.e. an object that presents roughly the same characteristics over a large range of scale, a core way of identifying fractal trend is the Box-Counting method of Schroeder (1991). This method uses the Dynamic time warping (DTW) process to measure the similarities between these series using the presence of runs.

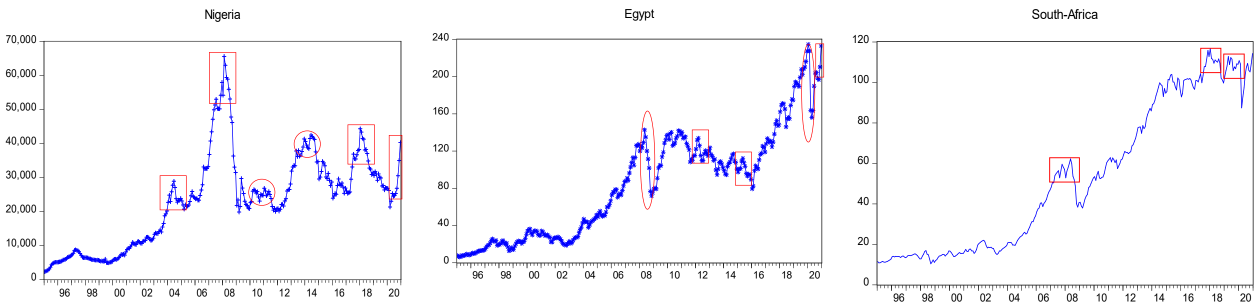

Figure 4.1: Monthly Trend of Stock Prices i.e. All Share Index in Nigeria, Egypt, and South Africa from 1995 to 2020.

Source: E-Views Version 11 Output.

Dynamic Time Warping Runs Test

Table 1: Dynamic Time Warping Runs Test for Nigeria, Egypt and South Africa's All Share Index Jan 1995 to Dec 2020.

	Nigeria	Egypt	South-Africa
Test Value[a]	23.1277761	84.42774	52.4922633
Cases < Test Value	144	151	165
Cases >= Test Value	168	161	147
Total Cases	312	312	312
Number of Runs	20	6	8
Z	-15.525	-17.124	-16.896
Asymp. Sig. (2-tailed)	.000	.000	.000
Fractal Trend	2	2	1

a. Mean

Source: SPSS Version 26 Output.

The study observed significant fractal trends in the various countries. On a specific note, the study observed;

i. In Nigeria, there were Six (6) iterations of fractal tendencies in two (2) variants.
ii. In Egypt, there were five (5) iterations of fractal trends in two (2) variants.
iii. While South Africa had the least display of fractal trends which was in three (3) iterations and only one type.
iv. This shows that in terms of efficiency, the fractal tendencies makes the Nigerian market the least efficient, followed by the Egyptian market, while South Africa shows the relatively most efficient market amongst the sampled firms.

Test of Weak Form Efficiency

Fractal Price Hypothesis being a strand of the weak form efficiency requires evidence of the existence of weak form efficiency which is popularly attributable to the serial correlation test. The two common residual tests of serial correlation are Correlograms Q-Statistics and the Breusch-Godfrey LM tests. The study therefore proceeds to present the Correlograms

Q-Statistics and the Breusch-Godfrey LM tests. These tests are dependent on lagged values. This therefore leads to the test of lag length as follows;

Lag Length Selection Test

In general, too many lags inflate the standard errors of coefficient estimates and thus imply an increase in the forecast error while omitting lags that should be included in the model may result in an estimation bias. To determine the best/maximum lag length to employ in estimating the autocorrelation test, the study employs the lag length selection estimation as presented below as follows;

Table 2: Lag Length Selection Estimation

VAR Lag Order Selection Criteria
Endogenous variables: EGYPT NIGERIA SOUTH_AFRICA
Exogenous variables: C
Sample: 1995M01 2020M12
Included observations: 304

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-6165.625	NA	8.46e+13	40.58306	40.61974	40.59773
1	-4327.792	3627.301	5.04e+08	28.55126	28.69799	28.60996
2	-4300.541	53.24655	4.47e+08	28.43119	28.68796	28.53391
3	-4282.105	35.66083	4.20e+08	28.36911	28.73592	28.51584
4	-4266.220	30.41112	4.01e+08	28.32381	28.80067	28.51457
5	-4243.440	43.16245	3.67e+08	28.23315	28.82005	28.46793
6	-4236.067	13.82402	3.71e+08	28.24386	28.94080	28.52265
7	-4230.106	11.05937	3.78e+08	28.26385	29.07084	28.58667
8	-4215.711	26.42243	3.65e+08	28.22836	29.14539	28.59519
9	-4647.971	63.37251	4.20e+08	28.36911	28.73592	28.51584
10	-4973.947	84.37462*	4.01e+08*	28.32381*	28.80067*	28.51457*

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Source: E-Views Version 11 Output.

Table 2 above shows that the maximum/optimal lag to employ in further estimation is lag 10. This is due to the ranked/starred values across the 10-lag row. The study therefore employed the 10th lag in other estimation. Based on the definition of the optimal lag, the study proceeded to the serial correlation test using the Correlograms Q-Statistics and the Breusch-Godfrey LM tests.

Serial Correlation Test

Serial correlation (also called Autocorrelation) is where error terms in a time series transfer from one period to another. Serial correlation is the relationship between a given variable and a lagged version of itself over various time intervals. It measures the relationship between a variable's current value given its past values. A variable that is serially correlated indicates that it may not be random. In other words, the error for one time period *a* is correlated with the error for a subsequent time period *b*. A stock price displaying positive serial correlation has a positive pattern. A security that has a negative serial correlation has a negative influence on itself over time.

Correlograms Q-Statistics

The Q-statistic is often used as a test of whether the series is white noise or has an observable pattern overtime.

Table 3: Nigeria, Egypt and South Africa Stock Price Correlograms Q-Statistics Output

Date: 09/27/21 Time: 11:55

Sample: 1995M01 2020M12

Included observations: 312

<table>
<tr><th></th><th></th><th></th><th colspan="3">Nigeria</th><th colspan="2">Egypt</th><th colspan="3">South Africa</th></tr>
<tr><th>Autocorrelation</th><th>Partial Correlation</th><th></th><th>AC</th><th>PAC</th><th>Q-Stat</th><th>AC</th><th>Q-Stat</th><th>AC</th><th>Q-Stat</th><th>Prob</th></tr>
<tr><td>.|*******</td><td>.|*******</td><td>1</td><td>0.983</td><td>0.983</td><td>304.35</td><td>0.980</td><td>302.39</td><td>0.991</td><td>309.33</td><td>0.000</td></tr>
<tr><td>.|*******</td><td>.|. |</td><td>2</td><td>0.964</td><td>-0.056</td><td>598.19</td><td>0.959</td><td>592.89</td><td>0.982</td><td>614.24</td><td>0.000</td></tr>
<tr><td>.|*******</td><td>*|. |</td><td>3</td><td>0.943</td><td>-0.090</td><td>880.01</td><td>0.941</td><td>873.67</td><td>0.974</td><td>915.23</td><td>0.000</td></tr>
<tr><td>.|*******</td><td>*|. |</td><td>4</td><td>0.918</td><td>-0.112</td><td>1147.9</td><td>0.925</td><td>1146.1</td><td>0.966</td><td>1212.3</td><td>0.000</td></tr>
<tr><td>.|******|</td><td>.|. |</td><td>5</td><td>0.894</td><td>0.050</td><td>1403.1</td><td>0.911</td><td>1410.6</td><td>0.959</td><td>1505.5</td><td>0.000</td></tr>
<tr><td>.|******|</td><td>*|. |</td><td>6</td><td>0.869</td><td>-0.068</td><td>1644.6</td><td>0.895</td><td>1667.2</td><td>0.951</td><td>1795.0</td><td>0.000</td></tr>
<tr><td>.|******|</td><td>.|. |</td><td>7</td><td>0.842</td><td>-0.022</td><td>1872.5</td><td>0.881</td><td>1916.2</td><td>0.944</td><td>2080.9</td><td>0.000</td></tr>
<tr><td>.|******|</td><td>.|. |</td><td>8</td><td>0.815</td><td>-0.057</td><td>2086.4</td><td>0.869</td><td>2159.7</td><td>0.937</td><td>2363.7</td><td>0.000</td></tr>
<tr><td>.|******|</td><td>.|. |</td><td>9</td><td>0.789</td><td>0.053</td><td>2287.6</td><td>0.860</td><td>2399.1</td><td>0.930</td><td>2643.6</td><td>0.000</td></tr>
<tr><td>.|******|</td><td>.|. |</td><td>10</td><td>0.762</td><td>-0.032</td><td>2476.1</td><td>0.853</td><td>2635.2</td><td>0.925</td><td>2921.4</td><td>0.000</td></tr>
</table>

Source: E-Views Version 11 Output.

Table 3 shows that the Q-statistics are significant at all lags as they possess probability value less than 0.05 (5%), indicating significant serial correlation in the residuals. It also shows Autoregressive Conditional Heteroskedasticity (ARCH) trend in the residuals. Judging by these results, this shows that Nigerian stock prices overtime are not random and that there is a significant relationship between current value of stock prices/all share index and its past values. This therefore upholds evidence of weak form efficiency and further reinforces our found fractal price trends in Nigeria, Egypt, and South Africa.

Breusch-Godfrey Serial Correlation LM Test

The Breusch–Godfrey test is a test for autocorrelation in the errors in a regression model. It makes use of the residuals from the model being considered in a regression analysis, and a test statistic is derived from these. The null hypothesis is that there is no serial correlation of any order up to the 10th lag as is in this case.

Table 4: Breusch-Godfrey Serial Correlation LM Test: Nigeria, Egypt, and South Africa

Breusch-Godfrey Serial Correlation LM Test: Nigeria

F-statistic	9.686708	Prob. F(5,307)	0.0001
Obs*R-squared	18.46086	Prob. Chi-Square(2)	0.0001

Breusch-Godfrey Serial Correlation LM Test: Egypt

F-statistic	21.45807	Prob. F(2,307)	0.0000
Obs*R-squared	38.14320	Prob. Chi-Square(2)	0.0000

Breusch-Godfrey Serial Correlation LM Test: South Africa

F-statistic	6.839872	Prob. F(2,307)	0.0328
Obs*R-squared	7.692370	Prob. Chi-Square(2)	0.0290

Source: E-Views Version 11 Output.

From table 4 above, it can be observed for Nigeria, Egypt and South Africa stock prices that the probability value of 0.001, 0.0000, and 0.0328 is seen to be less than the 0.05 (5%) threshold. This dictates for the study to rejects the hypothesis of no serial correlation up to order two. The Q-statistic and the LM test both indicate that the residuals are serially correlated and therefore verifies the evidence of a weak form efficiency.

Stationarity Test

A parameter that needs to be satisfied to test for long run effect of volatile variables such as stock and return is the stationarity test. The study therefore proceeds with the evaluating of employed variables stationarity at level as presented below in Table 5;

Table 4: Summary Compilation of Stationarity Test of Employed Variables at Level (0).

	ADF t-stat	Test Critical Values			Prob	Unit Root	Comment
		1% Level	5% Level	10% Level			
NASI	-2.148653	-3.451421	-2.870712	-2.571728	0.2259	Present	Not Stationary at Level i.e. 0(0).
NVOL	-1.882509	-3.451214	-2.870621	-2.571679	0.3404	Present	Not Stationary at Level i.e. 0(0).
NRTN	-14.11338	-3.451283	-2.870651	-2.571695	0.0000	Absent	Evidence of Stationarity at level
EASI	-0.024057	-3.451351	-2.870682	-2.571711	0.9548	Present	Not Stationary at Level i.e. 0(0).
EVOL	-2.297930	-3.452066	-2.870996	-2.571880	0.1733	Present	Not Stationary at Level i.e. 0(0).
ERTN	-13.36291	-3.451351	-2.870682	-2.571711	0.0000	Absent	Evidence of Stationarity at level
SASI	-0.112124	-3.451214	-2.870621	-2.571679	0.9662	Present	Not Stationary at Level i.e. 0(0).
SVOL	-0.744287	-3.452066	-2.870996	-2.571880	0.9929	Present	Not Stationary at Level i.e. 0(0).
SRTN	-13.99678	-3.451283	-2.870651	-2.571695	0.0000	Absent	Evidence of Stationarity at level

First Difference							
NASI	-6.389439	-3.451421	-2.870712	-2.571728	0.0000	Absent	Stationary at First Difference i.e. I(1)
NVOL	-17.55058	-3.451283	-2.870651	-2.571695	0.0000	Absent	Stationary at First Difference i.e. I(1)
NRTN	-	-	-	-	-	-	-
EASI	-12.59566	-3.451351	-2.870682	-2.571711	0.0000	Absent	Stationary at First Difference i.e. I(1)
EVOL	-3.911257	-3.452066	-2.870996	-2.571880	0.0052	Absent	Stationary at First Difference i.e. I(1)
ERTN	-	-	-	-	-	-	-
SASI	-16.49620	-3.451283	-2.870651	-2.571695	0.0000	Absent	Stationary at First Difference i.e. I(1)
SVOL	-4.555840	-3.452066	-2.870996	-2.571880	0.0035	Absent	Stationary at First Difference i.e. I(1)
SRTN	-	-	-	-	-	-	-

Where: **ADF** - Augmented Dickey Fuller.
Prob – Probability Level.

Source: EViews 11 Output
Table 4 above shows that the stock prices and liquidity (trading volume to gross domestic product) attained stationarity and lacked unit root when differenced. This can be observed as their test statistics values are observed to be greater than the absolute value of their respective test critical values at the 1, 5, and 10% level. This, therefore, shows that our employed variables have a reliable trend that would enable the further analysis to be free from spurious or unreliable outputs. In light of the observation of stationarity at level and first difference, the study would proceed to undertake the Autoregressive Conditional Heteroscedasticity due to the lack of constancy in the variance of the error term as reported in previous estimates.

Autoregressive Conditionally Heteroscedastic (ARCH/GARCH)
The goal of the study is to determine the relationship between volatile variables. This therefore mandates the need of the ARCH/GARCH estimation. ARCH models were introduced by Engle (1982) and generalized as GARCH (Generalized ARCH) by Bollerslev (1986) and Taylor (1986). These models are widely used in various branches of econometrics, especially in financial time series analysis. It helps to determine the nature of relationship between volatile trends and would be instrumental in answering the study's research questions and hypotheses.

Table 5: Volatility Relationship between Stock Prices and Liquidity in Nigeria
Dependent Variable: D(NVOL)
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 09/28/21 Time: 18:20
Sample: 1995M01 2020M12
Included observations: 312
Convergence achieved after 43 iterations
Coefficient covariance computed using outer product of gradients

Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	14.58422	1.593525	9.152175	0.0000
D(NASI)	5.485530	0.110708	49.54957	0.0000
Variance Equation				
C	27.66992	11.23746	2.462294	0.0138
RESID(-1)^2	1.050764	0.223972	4.691489	0.0000
GARCH(-1)	0.089758	0.090297	0.994027	0.3202

Egypt

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.813809	0.716745	1.135423	0.2562
D(NASI)	0.025829	0.025125	1.028030	0.3039
Variance Equation				
C	7.050072	2.621987	2.688828	0.0072
RESID(-1)^2	0.184806	0.065360	2.827510	0.0047
GARCH(-1)	0.667942	0.099996	6.679678	0.0000

South Africa

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	6.885583	0.017928	384.0639	0.0000
D(EASI)	-0.012747	0.000160	-79.42140	0.0000
Variance Equation				
C	0.010889	0.009346	1.165149	0.2440
RESID(-1)^2	1.729096	0.068230	25.34211	0.0000
GARCH(-1)	-0.003807	0.010946	-2.238708	0.0113

Source: E-Views Version 11 Output.

Autoregressive Conditional Heteroskedasticity. The study observed that;
In terms of the relationship between fractal prices and liquidity/volume of transactions, the study observed a significant relationship between stock prices and liquidity position of the various countries. On a specific note, the study observed a;

i. Positive (5.485530) and significant (0.0000 < 0.05) relationship between stock prices and liquidity in the Nigerian stock market. This shows that an increase in stock prices tend to lead to an increase in liquidity position or transaction volume i.e., a bullish behaviour in the Nigerian stock market and a decrease in stock prices tends to reduce the liquidity position in the Nigerian stock market therefore showing a bearish behaviour in the Nigerian stock market.

ii. Negative (-0.012747) and significant (0.0000 < 0.05) relationship between stock prices and liquidity in the Egyptian stock market. This means that an increase in stock prices tend to lead to a decrease in liquidity position or transaction volume i.e., a bearish behaviour in the Egyptian stock market and a decrease in stock prices tends to increase the liquidity position in the Egyptian stock market therefore showing a bullish behaviour in the Egyptian stock market.

iii. Positive (0.931463) and significant (0.0000 < 0.05) relationship between stock prices and liquidity in the Nigerian stock market. This shows that an increase in stock prices tend to lead to an increase in liquidity position or transaction volume i.e., a bullish behaviour in the South African stock market and a decrease in stock prices tends to reduce the liquidity position in the South African stock market therefore showing a bearish behaviour in the South African stock market.

Liquidity Spill-Over

Table 6: Volatility Relationship between Stock Prices and Returns in Egypt

Dependent Variable: D(ERTN)
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 09/28/21 Time: 18:21
Sample (adjusted): 1995M02 2020M12
Included observations: 311 after adjustments
Convergence achieved after 22 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	1.591791	0.713316	2.231537	0.0256
D(EASI)	-0.003631	0.007548	-0.480987	0.6305
Variance Equation				
C	5.875814	3.361259	1.748099	0.0804
RESID(-1)^2	0.084101	0.039792	2.113550	0.0346
GARCH(-1)	0.802171	0.099121	8.092865	0.0000

GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	19.84931	0.524659	37.83279	0.0000
D(SASI)	0.931463	0.010887	85.56047	0.0000
Variance Equation				
C	6.499377	1.491422	4.357839	0.0000
RESID(-1)^2	1.000049	0.244076	4.097286	0.0000
GARCH(-1)	-0.001082	0.049320	-0.021937	0.9825

GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	1.308752	0.448017	2.921208	0.0035
D(SASI)	-0.005152	0.007475	-0.689297	0.4906
		Variance Equation		
C	5.071561	2.284254	2.220227	0.0264
RESID(-1)^2	0.195919	0.084788	2.310684	0.0209
GARCH(-1)	0.534684	0.197170	2.711795	0.0067

Source: E-Views Version 11 Output.

In the light of the relationship between fractal prices and returns, the study observed that there is no significant link between fractal prices and returns made by investors. Specifically, the study observed a;

i. Positive (0.025829) and insignificant (0.3039 > 0.05) relationship between stock prices and investors returns in the Nigerian stock market. This shows that an increase in stock prices does not significantly lead to an increase in investors returns in the Nigerian stock market.
ii. Negative (-0.003631) and insignificant (0.6305 > 0.05) relationship between stock prices and investors returns in the Egyptian stock market. This means that an increase in stock prices does not significantly lead to a decrease in investors returns in the Egyptian stock market.
iii. Negative (-0.005152) and insignificant (0.4906 > 0.05) relationship between stock prices and investors returns in the Nigerian stock market. This shows that an increase in stock prices does not significantly lead to an increase in investors returns in the South African stock market.

Overall, the study observed that, while investors reacted to fractal and repetitive trends, they did not make significant gains from it. This therefore points at the futility of investors behavior in light with Mackay's (1841) popular article "Popular Delusions and the Madness of Crowds" that introduced the chronological timetable of various panics and schemes during the course of history by investors in the stock market. The study like this finding revealed how human behaviour shows itself in financial markets. This theory like the majority of the theories of economics and finance presume that investors behave rationally and always consider all information available in the decision making process (Jureviþienơ & Ivanova, 2013) but do not always get the desire outcome. During recent years, increasing fluctuations in the market cannot be explained referring to the hypothesis of rationality. Considering current trends in the finance markets, it can be stated that even the smartest investors experience psychological deviations and still observe adverse effect in the long run.

This means that, when investors observe fractal tendencies in the market, they all unanimously conform to rational expectation (conformist effect -when they adjust their behaviour to the norms in the particular group and so escape personal responsibility for the mistakes) and they trying to break even or make profit only to fall into the "snake bite effect" (an effect that emerges when having experienced huge losses, investors are afraid to take risks and avoid investing in riskier securities.). The common investment mistakes are directly linked with human psychology and internal state at the moment of investment. Investors usually do not consider information accuracy which, in turn, causes the mistakes while buying or selling stock and reduces the profits that could have been earned. Over-confidence as well as the lack of confidence can impede rational decisions.

Advocates of the efficient markets hypothesis (EMH) have long based their argument on the belief that stock markets are generally governed by the rational expectations of all market participants that predispose security prices to reflect all available information (Fama, 1965, 2013). All studies predicated on EMH have invariably yielded findings and conclusions that give the impression that the occurrence and persistence of crises is an aberration from the perceived norm of efficient and well-behaved financial markets and systems. One of the two hypotheses upon which the study is based is that financial crises especially those manifesting as either stock market crises or stock market corrections are an inescapable feature of modern financial systems.

5. CONCLUSION, AND RECOMMENDATIONS

Conclusions

Based on the findings of the study, the study concludes that there are fractal price trends in the Nigerian, Egyptian and South African markets. These prices are observed to affect liquidity and volume of transactions but do not affect investors returns. In terms of the relationship between fractal prices and liquidity/volume of transactions, the study observed a significant relationship between stock prices and liquidity position of the various countries. In the light of the relationship between fractal prices and returns, the study observed that there is no significant link between fractal prices and returns made by investors. Overall, the study has revealed that despite the rational information provided by a fractal market, careful and informed investor can yet lose the opportunity to earn profits. This therefore initiate the "snake bite" effect, causing the fear to take risks, have the influence on investor's further decisions and behaviour, and prevents him from profit lock which, in turn, has the impact on profit margin and investment return.

Recommendations

In light of the observed findings, it is recommended that;

i. In view of the shallow depth of the Egyptian and the Nigerian stock market, the breadth and width of the Nigerian and Egyptian capital market should be broadened. Perhaps, this will give investors more confidence to invest in shares and catalyze activities in these markets.
ii. The extent to which investors insignificantly earned from their trading shows that investors would be prudent to have some of their funds invested in both the stock market and government securities to hedge against potential losses due to exogenous crisis events.
iii. This indicates that even informed investors are prone to the wrath of the market which could be linked to external effect and shows that the economic bloc needs to put in place economic policy measures individually and collectively to partially insulate themselves from the repercussions of negative economic and financial events emanating from outside their economic boundaries.

iv. There is a need for the improvement of the mandate of multilateral financial institutions such as the International Monetary Fund (IMF), the Bank of International Settlements (BIS), the European Central Bank (ECB), and the Federal Reserve System (the Fed) so that they are more proactive in monitoring levels of liquidity in developed economies.
v. Advanced financial econometric modelling systems which taken into account the long memory features, volatility persistence, and entropy behaviour of developed and emerging economy stock markets would provide a scientific basis for such liquidity monitoring efforts. The research and analytical effort would need to be inter-disciplinary and cross-country in order to yield forecasts and projections of financial indices' behaviour that can be relied upon to craft effective and efficient policies.
vi. There is need for public enlightenment campaign to arouse the interest of inviting public who reside outside the major cities and who would wish to avail themselves the opportunities in both new issues and secondary market. It is also important for stockbrokers to behave responsibly in pricing and as much as possible avoid the temptation of push prices based on their portfolio holdings.

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